Phases of the Brans-Dicke Cosmology with Matter

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Abstract

We study the cosmology of the Brans-Dicke theory with perfect fluid type matter. In our previous work, we found exact solutions for any Brans-Dicke parameter ω and for general parameter γ of equation of state. In this paper we further study the cosmology of these solutions by analyzing them according to their asymptotic behaviors. The cosmology is classified into 19 phases according to the values of γ and ω. The effect of the cosmological constant to the Brans-Dicke theory is a particular case of our model. We give plot of time evolution of the scale factor by numerical investigations. We also give a comparison of the solutions for the theories with and without matter.

I. INTRODUCTION

Recent developments of the string theory suggest that in a regime of Planck length curvature, quantum fluctuation is very large so that string coupling becomes large and consequently the fundamental string degrees of freedom are not weakly coupled *good* ones [1]. Instead, solitonic degrees of freedom like p-brane or D-p-brane [2] are more important. Therefore it is a very interesting question to ask what is the effect of these new degrees of freedom to the space time structure especially whether including these degrees of freedom resolve the initial singularity, which is a problem in standard general relativity. The new gravity theory that can deal with such new degree of freedom should be a deformation of standard general relativity so that in a certain limit it should be reduced to the standard Einstein theory. The Brans-Dicke theory [4] is a generic deformation of the general relativity allowing variable gravity coupling. Therefore whatever is the motivation to modify the Einstein theory, the Brans-Dicke theory is the first one to be considered. As an example, low energy limit of the string theory contains the Brans-Dicke theory with a fine tuned deformation parameter (ω=-1).

Without knowing the exact theory of the p-brane cosmology, the best guess is that it should be a Brans-Dicke theory with matters. In fact there is a some evidence for this [5], where it is found that the natural metric that couples to the p-brane is the Einstein metric multiplied by certain power of dilaton field. In terms of this new metric, the action that gives the p-brane solution becomes Brans-Dicke action with definite deformation parameter ω depending on p. Using this action, Rama [6] recently argued that the gas of solitonic p-brane [5] treated as a perfect fluid type matter in a Brans-Dicke theory can resolve the initial singularity without any explicit solution. In a previous papers [7,8], we have studied this model and found exact cosmological solutions for any Brans-Dicke parameter ω and for general equation of state and classify the cosmology of the solutions according to the range of parameters involved.

In this paper we further study the cosmology of these solutions by analyzing them according to their asymptotic behaviors. The cosmology is classified into 19 phases according to the values of γ and ω. The effect of the cosmological constant to the Brans-Dicke theory is a particular case of our model. We give plot of time evolution of the scale factor by numerical investigations. We also give a comparison of the solutions for the theories with and without matter.

The rest of this paper is organized as follows. In section II, we set up the notation and review the result of our previous results [7,8]. In section III, we describe two new phases which were not mentioned in [7] and classify the cosmology into 19 phases. By the numerical work as well as the analytical method, the behaviors of the scale factor is presented explicitly by figures. In section IV, we summarize and conclude with some discussions.

II. BRANS-DICKE COSMOLOGY WITH MATTER

First, we briefly review our earlier work [7]. We consider the Brans-Dicke theory and analyze the evolution of the D dimensional homogeneous isotropic universe with the perfect fluid type matter. The action is given by

$$S = \int d^D x \sqrt{-g} e^{-\phi} \left[\mathcal{R} - \omega \partial_\mu \phi \partial^\mu \phi \right] + S_m, \tag{1}$$

where ϕ is the dilaton field and S_m is the matter part of the action. Here we assume that the matter has no dilaton coupling.

Let's assume that the matter can be treated as a perfect fluid with the equation of state

$$p = \gamma \rho, \gamma < 1. \tag{2}$$

Therefore our starting point is the equation of the Brans-Dicke theory [9,10]

$$\mathcal{R}_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{R} = \frac{e^\phi}{2} T_{\mu\nu} + \omega \{ \partial_\mu \phi \partial_\nu \phi - \frac{g_{\mu\nu}}{2} (\partial\phi)^2 \}$$
$$+ \{ -\partial_\mu \partial_\nu \phi + \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} \mathcal{D}^2 \phi - g_{\mu\nu} (\partial\phi)^2 \},$$
$$0 = \mathcal{R} - 2\omega \mathcal{D}^2 \phi + \omega (\partial\phi)^2 \tag{3}$$

where ϕ is the dilaton and $\mathcal{D}$ means a covariant derivative. $\mathcal{R}$ is the curvature scalar and the metric is given as the follows:

$$ds^2 = -\frac{1}{\mathcal{N}} dt^2 + e^{2\alpha(t)} \delta_{ij} dx^i dx^j \ \ (i,j = 1,2,\cdots,D-1), \tag{4}$$

where $e^{\alpha(t)}(= a(t))$ is the scale factor and $\mathcal{N}$ is the (constant) lapse function.

The energy-momentum tensor is given by

$$T_{\mu\nu} = p g_{\mu\nu} + (p + \rho) U_\mu U_\nu \tag{5}$$

where U_μ is the fluid velocity. The hydrostatic equilibrium condition of energy-momentum conservation is

$$\dot{\rho} + (D-1)(p+\rho)\dot{\alpha} = 0. \tag{6}$$

Using the equation of state, eq.(2), with a free parameter γ, we get the solution

$$\rho = \rho_0 e^{-(D-1)(1+\gamma)\alpha}, \tag{7}$$

where ρ_0 is a real number. Our goal is to study how the metric variables change their behavior for various values of γ and ω. Now, since we consider only the time dependence, the action can be brought to the following form

$$S = \int dt \ e^{(D-1)\alpha - \phi} \left[\frac{1}{\sqrt{\mathcal{N}}} \{ -(D-2)(D-1)\dot{\alpha}^2 + 2(D-1)\dot{\alpha}\dot{\phi} + \omega \dot{\phi}^2 \} \right.$$
$$\left. -\sqrt{\mathcal{N}} \rho_0 e^{-(D-1)(1+\gamma)\alpha + \phi} \right]. \tag{8}$$

where we eliminated p and ρ by eq.(6). After getting the constraint equation by varying over the constant lapse function, $\mathcal{N}$, we can set it to be 1.

Now, introducing a new time variable τ by

$$dt = e^{(D-1)\alpha - \phi} d\tau, \tag{9}$$

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and the new variables

$$X = -\frac{1}{2}[(D-1)(1-\gamma)\alpha - \phi],$$
$$Y = \alpha + \frac{\nu}{\kappa}X, \tag{10}$$

the action can be written as

$$S = \int d\tau \left[\frac{1}{\sqrt{\mathcal{N}}} \{(D-1)\kappa\dot{Y}^2 + \mu\dot{X}^2\} - \sqrt{\mathcal{N}}\rho_0 e^{-2X} \right], \tag{11}$$

where

$$\kappa = (D-1)(1-\gamma)^2(\omega - \omega_\kappa),$$
$$\nu = 2(1-\gamma)(\omega - \omega_\nu),$$
$$\mu = -\frac{4(D-2)}{\kappa}(\omega - \omega_{-1}),$$
$$\omega_\kappa = -\frac{D - 2D\gamma + 2\gamma}{(D-1)(1-\gamma)^2},$$
$$\omega_\nu = -\frac{1}{1-\gamma},$$
$$\omega_{-1} = -\frac{D-1}{D-2}. \tag{12}$$

The constraint equation is given by

$$0 = (D-1)\kappa\dot{Y}^2 + \mu\dot{X}^2 + \rho_0 e^{-2X}. \tag{13}$$

Note that for $D > 2$ and $\gamma < 1$, the sign of κ is determined by that of $\omega - \omega_\kappa$ and the sign of μ is determined by that of $\omega - \omega_{-1}$ and κ.

The equations of motion are simply

$$0 = \ddot{Y},$$
$$0 = \ddot{X} - \frac{\rho_0}{\mu}e^{-2X}. \tag{14}$$

When $\rho_0 = 0$, the situation is that of the string cosmology discussed first in [3] and the solution for X is $X = c\tau$. One can easily show that this solution has two disconnected branches in terms of the original time t; one is inflation-type and the other is FRW-type. If $\rho_0 \neq 0$, the asymptotic behavior of X is $X \sim c \mid \tau \mid$ as we will see later. In other word, the behavior of cosmology at $\rho_0 \neq 0$ is not continuously connected to that of cosmology at $\rho_0 = 0$ in $\tau \to -\infty$ region. Some of the new aspects of the cosmology due to the presence of the matter come from this discontinuity. If ρ_0 is a negative constant, then the solution oscillates in time, leading to unphysical solution. This includes the situation where there is a negative cosmological constant in the Brans-Dicke theory. In this paper, therefore, we will consider only positive ρ_0.

If ω is less than ω_{-1}, the kinetic term of the dilaton has a negative energy in Einstein frame. So we will consider the case where ω is larger than ω_{-1}. According to the sign of κ, the types of solutions are very different. When κ is negative, the exact solution is

$$X = \ln\left[\frac{q}{c}\cosh(c\tau)\right],$$
$$Y = A\tau + B, \tag{15}$$

where c, A, B and $q = \sqrt{\frac{\rho_0}{|\mu|}}$ are arbitrary real constants. Using the constraint equation, we can determine A in terms of other parameters,

$$A = \frac{c}{\delta}, \qquad \text{with} \quad \delta = \sqrt{-\frac{(\mathrm{D}-1)\kappa}{\mu}} = \frac{|\kappa|}{2\sqrt{1 + \omega\frac{\mathrm{D}-2}{\mathrm{D}-1}}}. \tag{16}$$

If κ is zero, it turns out that the solution of the equations of motion does not satisfy the constraint equation. If κ is positive, the solution is

$$X = \ln\left[\frac{q}{c}\,|\sinh(c\tau)|\,\right],$$
$$Y = \frac{c}{\delta}\tau + B. \tag{17}$$

III. PHASES OF THE BRANS-DICKE THEORY

In [7], the behaviors of the scale factor were classified by 16 phases according to ω and γ, see figure 1, and we showed that the asymptotic behaviors of scale factor $a(\tau)$ and time $t(\tau)$ at $\tau \to \pm\infty$ are as follows:

$$t - t_0 \approx \frac{1}{T_\pm}\left(e^{T_\pm\tau} - e^{T_\pm\tau_0}\right)$$
$$T_\pm = \frac{2c}{|\kappa|}\left[(D-1)\gamma\sqrt{1 + \omega\frac{D-2}{D-1}} \mp \mathrm{sign}(\kappa)\{\kappa + (D-1)\gamma(1 + \omega(1-\gamma))\}\right],$$
$$a(\tau) \approx e^{H_\pm\tau}$$
$$H_\pm = \frac{2c}{|\kappa|}\left[\sqrt{1 + \omega\frac{D-2}{D-1}} \mp \mathrm{sign}(\kappa)\{1 + \omega(1-\gamma)\}\right]. \tag{18}$$

Note that the range of t is determined by the sign of $T_\pm$:

$$(-\infty, \infty) \quad \text{if} \ \ T_- < 0 < T_+,$$
$$(-\infty, t_f) \quad \text{if} \ \ T_- < 0 \ \text{and} \ \mathrm{T}_+ < 0,$$
$$(t_i, \infty) \quad \text{if} \ \ T_- > 0 \ \text{and} \ \mathrm{T}_+ > 0,$$
$$(t_i, t_f) \quad \text{if} \ \ T_+ < 0 < T_-.$$

For $\kappa > 0$, $t(\tau)$ and $a(\tau)$ behave as [7]

$$t \approx -\mathrm{sign}(\tau)\frac{q^{-\eta}e^{(D-1)\gamma B}}{(\eta - 1)}\frac{1}{|\tau|^{\eta-1}},$$
$$a \approx e^{B}(q\,|\tau|)^{-\frac{2(1-\gamma)(\omega-\omega_\nu)}{|\kappa|}}, \tag{19}$$

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as τ goes to zero, where $\eta = 2 + \frac{(D-1)\gamma\nu}{\kappa}$. $t(\tau)$ is singular at $\tau \to 0$ if $\eta > 1$. So for $\kappa > 0$ and $\eta > 1$, the scale factor $a(t)$ has two branches. The asymptotic form of $a(t)$ as a function of t is given by

$$
\begin{aligned}
a(t) &\approx [T_-(t-t_i)]^{H_-/T_-} \quad \text{at} \ \ \tau \to -\infty, \\
a(t) &\approx [T_+(t-t_f)]^{H_+/T_+} \quad \text{at} \ \ \tau \to \infty,
\end{aligned}
\tag{20}
$$

where t_i (t_f) is a starting (ending) point at a finite time. Eq.(18) contains the cases where t starts from $-\infty$ and/or ends at ∞ by setting $t_i = 0$ and/or $t_f = 0$. According to the sign of $T_\pm$ and $H_\pm/T_\pm$ and the singularity at $\tau = 0$, we classified the behavior of the Brans-Dicke theory [7]. Here we summarize the result by table 1.

phase	sign of κ	sign of T_-	sign of T_+	range of t	sign of H_-/T_-	sign of H_+/T_+
I	-	+	-	$[t_i, t_f]$	+	+
II	-	-	-	$(-\infty, t_f]$	-	+
III^-	+	+		$[t_i, t_f]$	+	
III^+	+		+	$[t_i, \infty)$		+
IV	-	-	+	$(-\infty, \infty)$	+	+
V	-	+	+	$[t_i, \infty)$	-	+
VI	-	+	+	$[t_i, \infty)$	+	+
VII^-	+	+		$[t_i, \infty)$	+	
VII^+	+		+	$(-\infty, \infty)$		+
$VIII^-$	+	+		$[t_i, \infty)$	+	
$VIII^+$	+		-	$(-\infty, t_f]$		-
IX^-	+	+		$[t_i, \infty)$	+	
IX^+	+		+	$(-\infty, \infty)$		+
X^-	+	+		$[t_i, \infty)$	+	
X^+	+		-	$(-\infty, t_f]$		-
XI^-	+	+		$[t_i, \infty)$	+	
XI^+	+		-	$(-\infty, t_f]$		+

Table 1. The sign of $T_\pm$ determines the range of time t as following; $t(\tau)$ maps the real line of τ to (1) if $T_- < 0 < T_+$, $(-\infty, \infty)$ (2) if $T_- < 0$ and $T_+ < 0$, $(-\infty, t_f]$ (3) if $T_- > 0$ and $T_+ > 0$, $[t_i, \infty)$ (4) if $T_+ < 0 < T_-$, $[t_i, t_f]$. The sign of $H_\pm/T_\pm$ determines the asymptotic behavior of scale factor $a(t)$.

Now, notice that not only the sign of $H_\pm/T_\pm$ but also that of $H_\pm/T_\pm - 1$ is important because the the universe will accelerate if $H_\pm/T_\pm - 1 > 0$ and decelerate if $H_\pm/T_\pm - 1 < 0$ when $\tau \to \pm\infty$. Therefore, we further classify the phases of cosmology accordingly.

A. Case $\omega < \omega_\kappa$

1. $H_-/T_- > 1$

• For $T_- > 0$, the condition $H_-/T_- > 1$ is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} > \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}. \tag{21}$$

If $\gamma < 1/(D-1)$, the condition is automatically satisfied. If $\gamma > 1/(D-1)$, inequality (21) turns out to be reduced to $\omega > \omega_\kappa$, which is surprising. This means that there is no solution. Therefore among the regions I and VI which have $T_- > 0$, $H_- > 0$ and $\omega < \omega_\kappa$, only I satisfies $H_-/T_- > 1$.

- If $T_- < 0$, the condition $H_-/T_- > 1$ is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} < \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}, \tag{22}$$

 whose solution is $\gamma > 1/(D-1)$ and $\omega < \omega_\kappa$. Only IV satisfies conditions, $\omega < \omega_\kappa$, $T_- < 0$, $H_- < 0$ and $H_-/T_- > 1$.

2. $H_+/T_+ > 1$

- If $T_+ > 0$, the condition $H_+/T_+ > 1$ implies

$$\sqrt{1 + \omega \frac{D-2}{D-1}} < -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}, \tag{23}$$

 whose solution is given by $\gamma < 1/(D-1)$ and $\omega < \omega_\kappa$. There is no region satisfying $T_+ > 0$, $\gamma < 1/(D-1)$ and $\omega < \omega_\kappa$.

- If $T_+ < 0$, $H_+/T_+ > 1$ is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} > -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}, \tag{24}$$

 whose solution is $\gamma > 1/(D-1)$ or $\omega > \omega_\kappa$ for $\gamma < 1/(D-1)$. There is no region satisfying the conditions, $\omega < \omega_\kappa$, $T_+ < 0$ and $H_+/T_+ > 1$.

B. Case $\omega > \omega_\kappa$

1. $H_-/T_- > 1$

- For $T_- > 0$, the condition $H_-/T_- > 1$ is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} < -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}, \tag{25}$$

 whose solution is $\gamma < 1/(D-1)$ and $\omega < \omega_\kappa$. Therefore there is no solution satisfying the conditions, $\omega < \omega_\kappa$ and $H_-/T_- > 1$.

- If $T_- < 0$, the condition is given by

$$\sqrt{1 + \omega \frac{D-2}{D-1}} > -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}. \tag{26}$$

The solution is $\gamma > 1/(D-1)$ or $\omega > \omega_\kappa$ for $\gamma < 1/(D-1)$. Therefore, the solution is summarized by $\omega > \omega_\kappa$. But in the case $\omega > \omega_\kappa$, there is no region satisfying $T_- < 0$.

2. $H_+/T_+ > 1$

- For $T_+ > 0$, the condition $H_+/T_+ > 1$ is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} > \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}. \tag{27}$$

The solution is $\gamma < 1/(D-1)$ or $\omega > \omega_\kappa$ for $\gamma > 1/(D-1)$. So the solution is summarized by $\omega > \omega_\kappa$ like the last case. The regions III^+, VII^+, IX^+ have the solution satisfying $\omega > \omega_\kappa$, $T_+ > 0$ and $H_+/T_+ > 1$.

- If $T_+ < 0$, the above condition is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} < \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}, \tag{28}$$

whose solution is $\gamma > 1/(D-1)$ and $\omega < \omega_\kappa$. Therefore, there is no solution satisfying $\omega < \omega_\kappa$, because the solution $\omega < \omega_\kappa$ is inconsistent with the assumption $\omega > \omega_\kappa$.

3. The power behavior of scale factor at $\tau \to 0$

At $\tau \to 0$, $a(t)$ is given by

$$a(t) \approx E \times |t|^\Gamma \tag{29}$$

where Γ is given by

$$\Gamma = \frac{2(1-\gamma)(\omega - \omega_\nu)}{(\eta - 1)\kappa}$$

and a constant E becomes

$$E = [q(\eta - 1)]^{\frac{2(1-\gamma)(\omega-\omega_\nu)}{(\eta-1)|\kappa|}} e^{B\left[1 - \frac{2(D-1)\gamma(1-\gamma)(\omega-\omega_\nu)}{(\eta-1)\kappa}\right]}.$$

For $\omega > \omega_\kappa$ and $\eta > 1$, the condition that Γ is positive is satisfied in the region $\omega > \omega_\nu$. The condition $\Gamma > 1$ is reduced to

$$(1 - \gamma) + [(D-1)\gamma + (D-3)]\omega + (D-2) < 0. \tag{30}$$

This gives the following solution

$$\omega < -\frac{D-2}{(1-\gamma)((D-1)\gamma+D-3)} \ \text{ for } \ \gamma > -\frac{D-3}{D-1}, \tag{31}$$

$$\omega > -\frac{D-2}{(1-\gamma)((D-1)\gamma+D-3)} \ \text{ for } \ \gamma < -\frac{D-3}{D-1}. \tag{32}$$

Note that for $\gamma > -\frac{D-3}{D-1}$, ω_κ is always greater than $-\frac{D-2}{(1-\gamma)((D-1)\gamma+D-3)}$. So in that case, there is no solution. As a result, the solution to $\Gamma > 1$ is given by eq.(32). This divide the region XI of figure 1 into two region: for $0 < \Gamma < 1$ we call this as region XI and for $\Gamma > 1$ we call this as region XII. See figure 2.

Now we summarize all possible phases in Table 2.

phase	sign of κ	sign of T_-	sign of T_+	range of t	H_-/T_-	Γ $(\tau \to 0)$	H_+/T_+
I	-	+	-	$[t_i, t_f]$	$H_-/T_- > 1$		$0 < H_+/T_+ < 1$
II	-	-	-	$(-\infty, t_f]$	$H_-/T_- < 0$		$0 < H_+/T_+ < 1$
III^-	+	+		$[t_i, t_f]$	$0 < H_-/T_- < 1$		
III^+	+		+	$[t_i, \infty)$			$H_+/T_+ > 1$
IV	-	-	+	$(-\infty, \infty)$	$H_-/T_- > 1$		$0 < H_+/T_+ < 1$
V	-	+	+	$[t_i, \infty)$	$H_-/T_- < 0$		$0 < H_+/T_+ < 1$
VI	-	+	+	$[t_i, \infty)$	$0 < H_-/T_- < 1$		$0 < H_+/T_+ < 1$
VII^-	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$\Gamma < 0$	
VII^+	+		+	$(-\infty, \infty)$		$\Gamma < 0$	$H_+/T_+ > 1$
$VIII^-$	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$\Gamma < 0$	
$VIII^+$	+		-	$(-\infty, t_f]$		$\Gamma < 0$	$H_+/T_+ < 0$
IX^-	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$0 < \Gamma < 1$	
IX^+	+		+	$(-\infty, \infty)$		$0 < \Gamma < 1$	$H_+/T_+ > 1$
X^-	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$0 < \Gamma < 1$	
X^+	+		-	$(-\infty, t_f]$		$0 < \Gamma < 1$	$H_+/T_+ < 0$
XI^-	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$0 < \Gamma < 1$	
XI^+	+		-	$(-\infty, t_f]$		$0 < \Gamma < 1$	$0 < H_+/T_+ < 1$
XII^-	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$\Gamma > 1$	
XII^+	+		-	$(-\infty, t_f]$		$\Gamma > 1$	$0 < H_+/T_+ < 1$

Table 2. All possible phases are classified. Here phases XII^- and XII^+ are new phases.

By the numerical work, the explicit scale factor behaviors of all phases are shown in following figures. Here we set $D = 4$.

All phases in figure 3 have no singularity at $\tau = 0$ and the phases II and IV have no initial singularity. The asymptotic behavior of scale factor is determined by $H_\pm/T_\pm$. Note that phases III^- and III^+, which are continuously connected at $\tau = 0$, are not distinguished in [7]. Since scale factor of phases $III^\pm$ vanishes at $\tau = 0$, they are divided into two phases in this paper.

Each region included in figure 4 and figure 5 has two branches and each branch defines a different phase. For example, the earlier asymptotic behavior of VII^- phase (figure 4a) and the later asymptotic behavior of VII^+ phase (figure 4b) are determined by $H_\pm/T_\pm$, the later behavior of VII^- phase and the earlier behavior of VII^+ phase are determined by the behavior of scale factor near $\tau \to \pm 0$. The phases, which have no initial singularity, are + phases in figure 4.

In figure 5, phases $XI^\pm$ and $XII^\pm$ are divided by Γ which is given by eq.(29) at $\tau = 0$; $0 < \Gamma < 1$ for phases $XI^\pm$ and $\Gamma > 1$ for phases $XII^\pm$.

IV. DISCUSSION AND CONCLUSION

In this paper we studied the effect of the matter in the Brans-Dicke cosmology. This was motivated [7] by the string cosmology [3] with gas of solitonic p-brane by treating them as a perfect fluid type matter in Brans-Dicke theory. In Brans-Dicke theory, the matter has no dilaton coupling. From the string theory point of view, this means that the matter has R-R charge. So the matter considered here corresponds to D-brane gas in string theory context. With this matter, we found exact cosmological solutions for any Brans-Dicke parameter ω and for general constant γ and classified all possible phases of the solutions according to the parameters involved. There are new two phases $XII^\pm$ different from $XI^\pm$ for the behavior of scale factor at $\tau = 0$. So the number of total phases is 19 and some of them have no initial singularity. We studied all the phases of cosmology numerically and gave the figures for the time evolution of the scale factor.

Recently, ref. [11] argued that Holographic principle in the presence of a cosmological constant might imply the absence of initial singularity. In the Brans-Dicke cosmology, we do find some solutions avoiding the initial singularity. However, when we regard the Brans-Dicke theory as a string cosmology, we might ask whether there are solutions which resolve the initial singularity and the graceful exit problem at the same time. However, the cosmological constant in Brans-Dicke theory is not a cosmological constant in string theory where the cosmological constant couples to the dilaton. To discuss the problem in our frame work, we have to consider the matter coupled with dilaton. We will discuss the problem in later publications.

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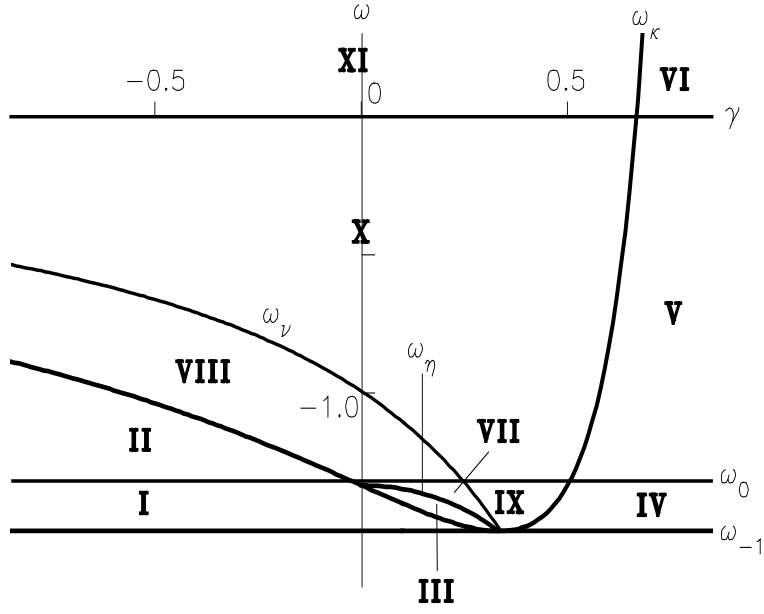
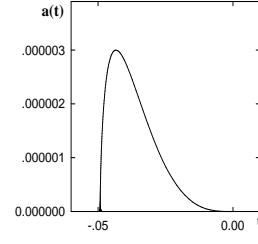FIG. 1.   Phase diagram of 11 regions
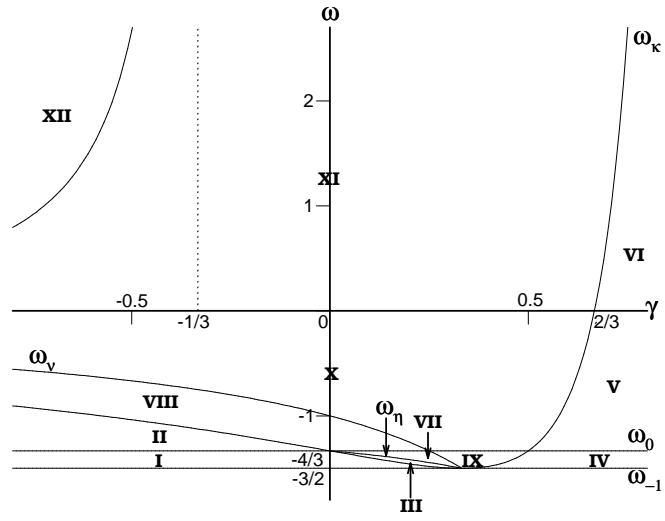
FIG. 2. Phase diagram, 12 regions
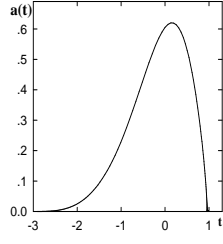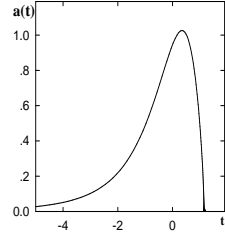
(a) phase I (b) phase II (c) phase III^- (d) phase III^+
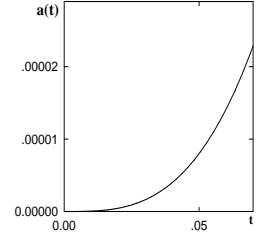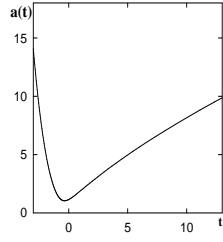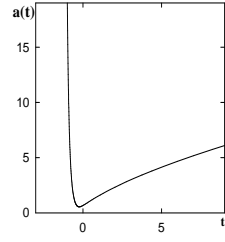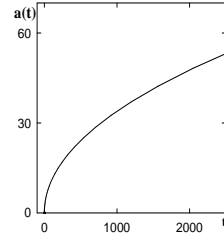

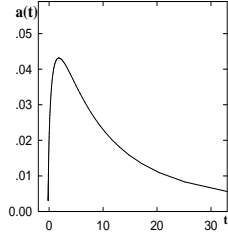
(e) phase IV (f) phase V (g) phase VI
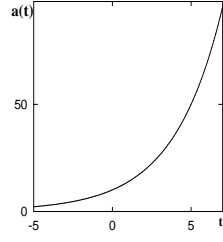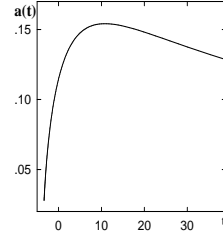
FIG. 3.   The behavior of the scale from phase I to VI
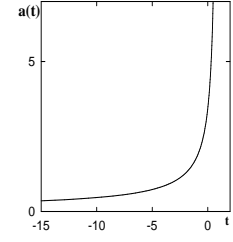
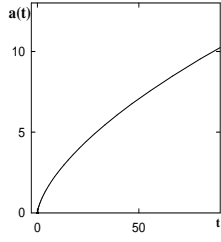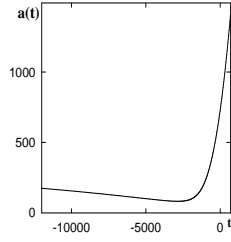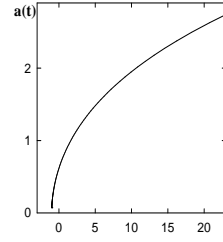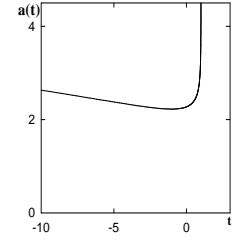(a) phase VII^- (b) phase VII^+ (c) phase $VIII^-$ (d) phase $VIII^+$

(e) phase IX^- (f) phase IX^+ (g) phase X^- (h) phase X^+

FIG. 4.   The behavior of scale factor from phase VII^- to X^+

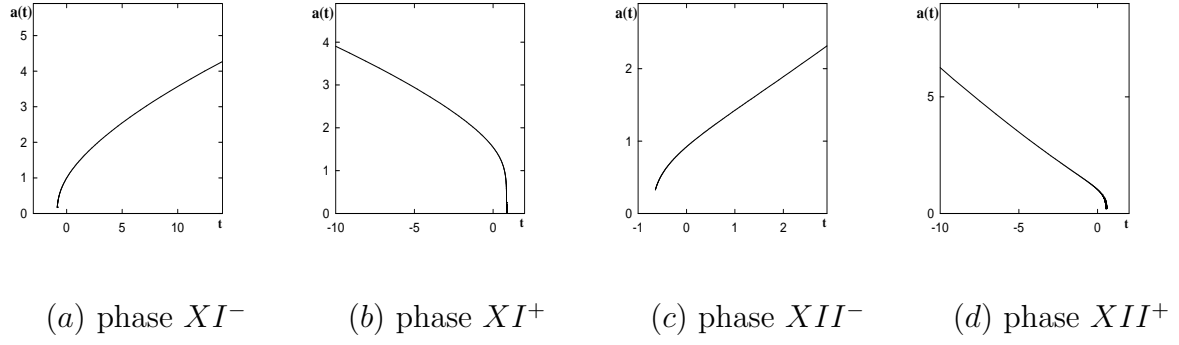(a) phase XI^- (b) phase XI^+ (c) phase XII^- (d) phase XII^+

FIG. 5.   The behavior of scale factor from phase XI^- to XII^+